High Energy Cosmic Neutrinos and the Equivalence Principle

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Abstract

Observation of the ultra-high energy neutrinos, in particular, detection of ν_τ from cosmologically distant sources like active galactic nuclei (AGN) opens new possibilities to search for neutrino flavor conversion. We consider effects of violation of the equivalence principle (VEP) on propagation of these cosmic neutrinos. Two effects are studied: (1) the oscillations of neutrinos due to the VEP in the gravitational field of our Galaxy and in the intergalactic space, (2) the resonance flavor conversion driven by the gravitational potential of the AGN. We show that the ultra-high energies of the neutrinos as well as cosmological distances to the AGN, or strong AGN gravitational potential will allow one to improve the accuracy of test of the equivalence principle by 21 orders of magnitude for massless or degenerate neutrinos ($\Delta f \sim 10^{-41}$)

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and at least by 12 orders of magnitude for massive neutrinos ($\Delta f \sim 10^{-28} \times (\Delta m^2/1\mathrm{eV}^2)$). Experimental signatures of the transitions induced by the VEP are discussed.

I. INTRODUCTION

Cosmologically distant objects such as the active galactic nuclei (AGN) can be intense sources of high-energy neutrinos [1]. The flux of these cosmic neutrinos is flavor non-symmetric. Models predict that the τ - neutrino flux is at least 3 orders of magnitude smaller than the fluxes of the electron and muon neutrinos. This opens a unique possibility of searching for effects of neutrino flavor transitions over intergalactic distances.

It was suggested recently [2] that large deep underwater and ice neutrino detectors will be able to identify the events induced by τ neutrinos with energies above 1 PeV. The τ-neutrinos produce the characteristic double-bang events. The first bang comes from the charged current interaction of the τ neutrino and the second one originates from the hadronic decay of the τ-lepton. At PeV energies the τ-lepton tracks have typical lengths of the order of 100 m and the two bangs are clearly separable. Moreover, a selection criterion of greater energy of the second bang in comparison with the first one makes the events to be essentially background-free [2], thereby guaranteeing the unambiguous detection of the τ-neutrinos.

It was estimated that the experiment is sensitive to the transition probabilities greater than

$$P \geq (3-5) \cdot 10^{-3}. \tag{1}$$

The transition can be induced by the vacuum oscillations [2]. Apart from the appearance of the relatively large ν_τ-flux one expects also a modification of number of the ν_e and ν_μ events. Cosmological distances to the AGN will allow one to probe the range of oscillation parameters: $\Delta m^2 > 10^{-15} \text{eV}^2$, $\sin^2 2\theta > 2P \approx 10^{-2}$. In particular, strong effect is expected for the values of parameters needed to solve the atmospheric neutrino problem [3].

In this paper we will consider the flavor transitions of the high-energy cosmic neutrinos induced by possible non-universality in the gravitational couplings of neutrinos. We show that the ultra-high neutrino energies and cosmological distances, or strong AGN gravitational potential, conspire to lead to great sensitivity in exploring the tiny non-universality.

The non-universality of the gravitational couplings of neutrinos can lead to the neutrino flavor oscillations [4]. Search for the neutrino oscillations in accelerator neutrino experiments and solar neutrino observation turns out to be a powerful tool of exploring possible violation of Einstein's equivalence principle [5–12]. Moreover, the oscillations and neutrino conversion induced by violation of the equivalence principle supply viable mechanism which could solve the solar and the atmospheric neutrino problems simultaneously [7,12].

The sensitivities to non-universality achievable in these methods vary depending upon the experimental means, on the presence or absence of matter effect, and on values of neutrino masses. Existing neutrino accelerator experiments restrict the non-universality parameter Δf up to the level of 10^{-15} [7,11,12], assuming that the supercluster's gravitational potential is of order 10^{-5} (see below.) It will be possible to improve it in the planned long baseline accelerator experiments which can reach the sensitivity 10^{-18} [6,7].

In the case of massless (or degenerate) neutrinos studies of the solar neutrinos can be sensitive to 10^{-20} [12]. This limit corresponds to the vacuum long length oscillations. In the case of the resonant flavor conversion of massive neutrinos due to the matter effect in the solar interior the sensitivities are at most $10^{-15} - 10^{-16}$ [9,10,12].

It has also been discussed that for $\Delta f > 4 \cdot 10^{-14}$ the conversion induced by VEP leads to substantial effect on supernova dynamics [12]. The arrival time difference between neutrinos and photons from SN1987a gives a modest bound of the order of 10^{-3} [13].

We may conclude that by using any neutrino sources and any experimental devices so far considered it appears difficult to go far beyond the sensitivity of $\sim 10^{-20}$.

In this paper we will show that observation of the ultra-high energy neutrinos from cosmologically distant sources will allow one drastically improve the accuracy of testing the equivalence principle. The paper is organized as follows. In Sec. II we will consider the gravity-induced oscillations of massless (or degenerate) neutrinos and estimate a sensitivity to violation of the equivalence principle (VEP). In Sec. III the gravity effects in the presence of nonzero neutrino masses and vacuum mixing are discussed. In particular, we describe the resonance flavor conversion driven by the gravitational potential. In Sec. IV we will apply

the results to neutrinos from the AGN. In Sec. V the experimental signatures of the VEP effects are considered. In Sec. VI we summarize the results.

II. GRAVITATIONALLY INDUCED OSCILLATIONS OF NEUTRINOS FROM THE AGN

Let us restrict ourselves to the two-flavor case, (ν_μ, ν_τ), for simplicity. According to the hypothesis of violation of the equivalence principle (VEP) [4] the flavor eigenstates ν_μ and ν_τ are the mixtures of the gravity eigenstates, ν_{2g}, ν_{3g}, whose gravitational couplings $f_2 G$ and $f_3 G$, where G is the Newton constant, are different $f_2 \neq f_3$. Evidently, $f_i \neq 1$ at least for one neutrino. Introducing gravitational mixing angle, θ_g, one can write

$$\nu_\mu = \cos\theta_g \nu_{2g} + \sin\theta_g \nu_{3g}, \quad \nu_\tau = -\sin\theta_g \nu_{2g} + \cos\theta_g \nu_{3g} . \tag{2}$$

The non-universality of the gravitational couplings can be parametrized as

$$\Delta f = \frac{f_3 - f_2}{\frac{1}{2}(f_3 + f_2)} . \tag{3}$$

The ν_{2g} and ν_{3g} neutrinos feel gravitational fields with slightly different strengths. This leads to a difference in the energies of the eigenstates (i.e. to the level energy splitting):

$$V_g \equiv \frac{1}{2}\Delta f E \Phi(x), \tag{4}$$

where E is the energy of neutrino, and $\Phi(x) = MG/r$ is the gravitational potential at distance r from an object of mass M in the Keplerian approximation. The energy level splitting induces a relative phase difference between wave functions of ν_{2g} and ν_{3g} which results in neutrino flavor oscillations in the same fashion as in the mass-induced case.

Let us first suggest that neutrinos are massless or have equal masses. As we will show later the matter effect is negligibly small. In this case the propagation of neutrinos has a character of oscillations with the depth fixed by θ_g and with the length, l_g, determined by V_g:

$$l_g = \frac{2\pi}{\Delta f E \Phi(x)} \ . \tag{5}$$

The oscillation probability can be written as

$$P = \sin^2 2\theta_g \sin^2 \left(\Delta f E \int \Phi(x) dx \right) \ . \tag{6}$$

The distinctive feature of (5) is that the oscillation length is inversely proportional to the neutrino energy, in contrast with the vacuum oscillation length which is directly proportional to E. Therefore the sensitivity to Δf increases with the neutrino energy and with the path-length in the gravitational field. Both factors are present for neutrinos from the AGN. The AGN are believed to produce high energy neutrinos with spectrum extended to $E \sim 10$ PeV, [1] and typical distances from our Galaxy to the AGN are $L_{AGN} \sim 100$ Mpc.

Let us estimate a sensitivity to Δf. For this purpose we should find the integral $I = \int \Phi(x) dx$ along the neutrino trajectory which appears in (6). The intergal has three contributions:

$$I = I_{AGN} + I_{IG} + I_G, \quad I_i \equiv \int \Phi_i(x) dx, \ (i = AGN, \ IG, \ G) \ , \tag{7}$$

where $\Phi(x)_{AGN}$, $\Phi(x)_{IG}$, and $\Phi(x)_G$ are the potentials created by the AGN itself, by all bodies in the intergalactic space and by our Galaxy, respectively.

We get $I_{AGN}(r) \approx -\frac{1}{2} R_S \log(r/R_e)$ for a radial trajectory, where R_S is the Schwarzschild radius of AGN: $R_S \simeq 3 \times 10^{11} (M_{AGN}/10^8 M_\odot)$ m and $R_e \approx (10 - 10^2) R_s$ is the radius of the neutrino emission region. Using $M_{AGN} \sim 10^8 M_\odot$ as a typical mass of the AGN, we find $R_S \sim 3 \times 10^{11}$ m and $R_e \sim 10^{13}$ m , and consequently

$$I_{AGN} \sim 10^{-10} \text{Mpc}. \tag{8}$$

In the intergalactic space the effect is dominated by the gravitational field of the so called Great Attractor [14]. This supercluster is located at the distance (43.5 ± 3.5) h_0^{-1} Mpc from the Earth, where h_0 is in the range 0.5-1.0. The mass of the Great Attractor is about $M_{sc} \sim 3 \times 10^{16} h_0^{-1} M_\odot$, where $M_\odot$ is the solar mass. [15]. In the Keplerian approximation the gravitational potential of this supercluster can be estimated as

$\Phi(R) = -5.2 \times 10^{-6} (R/100 Mpc)^{-1} (M_{sc}/10^{16} M_\odot)$. (Note that the weak-field approximation still applies). Then the integral I_{IG} satisfies the inequality

$$|I_{IG}| \gtrsim 5.2 \times 10^{-4} \left(\frac{L}{100 \text{Mpc}} \right) \left(\frac{M_{sc}}{10^{16} M_\odot} \right) \text{ Mpc} \tag{9}$$

for any trajectory of length L within radius of 100 Mpc.

The contribution of our Galaxy equals to $I_G = -G M_G \log (L_{AGN}/r)$ for radial trajectory. Using $M_G \sim 10^{11} M_\odot$ and $r = 10$ kpc for the mass and the radius of the Galaxy, respectively, and $L_{AGN} = 100$ Mpc we get $I_G \simeq -10^{-7}$ Mpc.

The supercluster gravitational effect dominates: $I \approx I_{IG}$. The reason is that the path-length in the gravitational field tends to cancel the inverse distance dependence of the Keplerian potential. Therefore, the oscillation probability is essentially governed by the mass of source of the gravitational field and the mass relation $M_{sc} \gg M_G \gg M_{AGN}$ leads to $I_{sc} \gg I_G \gg I_{AGN}$.

It is straightforward to make an order-of-magnitude estimation of the sensitivity. For $E = 1$ PeV and I_{IG} (at $\Phi = 10^{-5}$, and the distance $L = 100$ Mpc) one gets

$$|I_{IG} E| > |\Phi E L| = 1.5 \times 10^{41} \tag{10}$$

which means according to (6) that the phase of oscillation of order unity will be obtained for $\Delta f > 10^{-41}$.

If the great attractor is a fake object, then the dominant effect would be due to the gravitational field of our Galaxy: $I \approx I_G$. In this case the expected sensitivity to Δf is $\sim 10^{-37}$ which still implies an improvement by more than 17 orders of magnitude. This Δf can be considered as the conservative estimation of the sensitivity.

Let us show that in spite of cosmological distances the matter effect on the neutrino conversion can be neglected. Consider the $\nu_e - \nu_\tau$ system for which matter effect appears in the first order in the weak interactions. The cosmological baryon density estimated from the nucleosynthesis is $\rho_B \sim 10^{-31}$ g/cm^3. This gives the width of matter in the inter-galactic space: $d_{IG} \equiv \rho_B \cdot L_{AGN} \sim 3 \cdot 10^{-5}$ g/cm^2. According to spheroid-dark corona

models [16] the matter density of our Galaxy is $\rho \sim (1-10) \times 10^{-25}$ g/cm^3. This leads to the width $d_G \sim 3 \cdot (1-10) \times 10^{-3}$ g/cm^2. Finally, the width of matter crossed by neutrinos in the AGN is estimated as $d_{AGN} \sim (10^{-2} - 10^{-1})$ g/cm^2 [17]. Thus total width, $d_{total} \approx d_{AGN} \sim (10^{-2} - 10^{-1})$ g/cm^2, is much smaller than the effective width $d_0 \equiv \sqrt{2}\pi m_N/G_F \approx 2 \cdot 10^9$ g/cm^2 needed for appreciable matter effect. For $\nu_\mu - \nu_\tau$ channel the matter effect appears in high order of perturbation theory and the required effective width is even larger.

III. GRAVITATIONALLY INDUCED TRANSITIONS IN THE PRESENCE OF NEUTRINO MASSES

Most probably neutrinos are massive and mixed. The gravitational effects themselves can generate via the nonrenormalizable interactions the neutrino masses of the order $v^2/M_P \sim 10^{-5}$ eV [18], where v is the electroweak scale and M_P is the Planck mass. Moreover, there are some hints from solar, atmospheric, cosmological as well as accelerator data that neutrino masses are even larger than that value. Forthcoming experiments will be able to check the hints. In this connection we will consider the gravitational effects in the presence of neutrino masses and mixing, assuming that the latter will be determined from the forthcoming experiments.

In the presence of the vacuum and gravity mixing the effective Hamiltonian of neutrino system in the flavor basis $\nu = (\nu_\mu, \nu_\tau)$ reads [4] (for notation we follow [9]):

$$H(x) = \delta \begin{pmatrix} -c & s \\ s & c \end{pmatrix} + V_g \begin{pmatrix} -c_g & s_g \\ s_g & c_g \end{pmatrix}, \tag{11}$$

where $\delta \equiv \Delta m^2/4E$ with $\Delta m^2 \equiv m_3{}^2 - m_2{}^2$, $c \equiv \cos 2\theta$, and $c_g \equiv \cos 2\theta_g$, $etc.$. As we have shown in Sec. II the matter effects can be neglected. For antineutrinos the VEP may differ from that for neutrinos: $\Delta\bar{f} \neq \Delta f$, so that one may expect a difference of the effects in neutrino and antineutrino channels. We assume at the moment that all the parameters

of the Hamiltonian (11) are real. In general, the complex phase can be introduced in (11) which has some physical consequences [12]. We will discuss effects of the phase at the end of this section.

According to (11) the mixing angle in medium, θ_m, is fixed by

$$\tan 2\theta_m = \frac{s\delta + V_g s_g}{c\delta + V_g c_g} \ . \tag{12}$$

Evidently, for $V_g \gg \delta$ the mixing is determined by gravity: $\theta_m \approx \theta_g$. And for $V_g \ll \delta$ the mixing angle equals the vacuum angle: $\theta_m \approx \theta$.

As follows from (12) the mixing angle is zero at

$$V_g^0 = -\delta \frac{s}{s_g} \ . \tag{13}$$

If $\theta_g = \theta$ one has from (12) $\tan 2\theta_m = \tan 2\theta$. In this case the evolution of neutrino state is reduced to oscillations with the constant depth, $\sin^2 2\theta$, and the oscillation length

$$l_\nu = \frac{2\pi}{\delta + V_g} \ . \tag{14}$$

The phases due to mass difference and the gravitational effect add up. The gravitational phase dominates if $V_g > \delta$, or explicitly, if $\Delta f > \Delta m^2/(2E^2\Phi)$. For supercluster potential, $\Phi_{sc} \sim 10^{-5}$, and $E = 1$ PeV we get from this inequality $\Delta f > 5 \times 10^{-36}, 5 \times 10^{-28}, 5 \times 10^{-25}$ for $\Delta m^2 = 10^{-10}, 10^{-2}, 10$ eV^2, respectively. However, in all these cases the oscillations are averaged and even if Δm^2 will be known it is impossible to identify the gravity effects.

If $\theta \neq \theta_g$, the Hamiltonian (11) can lead to the resonance flavor conversion due to change of V_g with distance, i.e. to the gravity version of the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [19]. Note that apart from a brief remark in [9] the consideration in the existing literature is restricted to the level crossing driven by matter density change with distance. Here we describe the level crossing driven by gravitational potential change rather than matter density change.

As follows from (11) the resonance condition (condition for maximal mixing) is

$$V_g = -\frac{c\delta}{c_g} , \qquad (15)$$

and the resonance value of the potential equals

$$\Phi_R = -\frac{\Delta m^2}{2E^2 \Delta f} \frac{\cos 2\theta}{\cos 2\theta_g} . \qquad (16)$$

Correspondingly, V_g in the resonance is $V_g^R = \Delta f E \Phi_R / 2$.

The adiabaticity condition in the resonance reads [9,10]

$$\left| \frac{(\delta \sin 2\theta + V_g \sin 2\theta_g)^2}{\frac{d}{dr} V_g \cos 2\theta_g} \right|_{resonance} \gg 1. \qquad (17)$$

This condition simplifies under the Keplerian approximation. Substituting in (17) $dV_g/dr = -V_g/r$ and using the resonance condition (15) we can rewrite (17) as

$$(\tan 2\theta - \tan 2\theta_g)^2 \cos 2\theta_g \gg \frac{1}{r_R \delta} , \qquad (18)$$

where r_R is the radius at which the resonance condition is fulfilled: $r_R = r_R(\Delta f)$.

The conditions (15, 18) determine the sensitivity regions of parameters Δf, θ_g. Indeed, for fixed Δm^2 and $c \approx c_g \approx 1$ the minimal and maximal values of the potential Φ determine via the resonance condition the range of Δf for which the resonance neutrino conversion may take place. Maximal value of r_R at which the resonance condition is fulfilled gives the lower bound on mixing angles through the adiabaticity condition (18). If the adiabaticity condition is satisfied with vacuum mixing angle alone, i.e., $\sin^2 2\theta \approx \tan^2 2\theta \gg (r_R \delta)^{-1}$, then θ_g can be arbitrarily small. In this case the role of the gravitational effect is just to split levels. On the contrary, for $\theta = 0$ one has the lower bound on the gravitational mixing

$$\sin^2 2\theta_g \gg \frac{1}{r_R \delta} . \qquad (19)$$

For fixed mixing angles the adiabaticity condition can be rewritten as the lower bound on Δf. Indeed, substituting δ in the resonance condition (15) into (18) we find

$$\Delta f \gg \frac{2 \times 10^{-33}}{(\tan 2\theta - \tan 2\theta_g)^2 \cos 2\theta_g} \left(\frac{E}{1\text{PeV}} \right)^{-1} \left(\frac{M_{AGN}}{10^8 M_\odot} \right)^{-1} . \qquad (20)$$

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If the adiabaticity condition is fulfilled, then the transition probability is determined by the initial and final values of mixing angle:

$$P_a = \frac{1}{2} \left(1 - \cos 2\theta_{mi} \cos 2\theta_{mf}\right) \ . \tag{21}$$

In this connection let us consider a dependence of mixing angle θ_m on the potential Φ or level splitting $V_g = \Delta f \Phi E / 2$. The angle θ_m as the function of Φ crucially depends on the sign of $\Delta m^2 \Delta f$, and on whether $\theta_g > \theta$ or $\theta_g < \theta$. We focus first on the resonant channel, $\Delta m^2 \Delta f > 0$.

1). $\theta_g < \theta$. In this case one has $|V_g^0| > |V_g^R|$, i.e. the value of potential which corresponds to zero mixing (13) is bigger than the resonance value. For the initial splitting $|V_g^i| \gg |V_g^R|$, the gravity mixing dominates and $\theta_m \approx \theta_g + \pi/2$. With diminishing $|V_g|$, the angle θ_m decreases and mixing becomes zero ($\theta_m = \pi/2$) at $V_g = V_g^0$; then θ_m crosses the resonance value, $\theta_m = \pi/4$, and for $|V_g| \ll |\delta|$ approaches $\theta_m = 0$.

2). $\theta_g > \theta$. Now $|V_g^0| < |V_g^R|$. At $|V_g^i| \gg |V_g^R|$ one has $\theta_m \approx \theta_g - \pi/2$. With diminishing $|V_g|$ the angle θ_m increases and crosses resonance value $\theta_m = -\pi/4$. At $V_g = V_g^0$ the angle θ_m vanishes so that $\sin^2 2\theta_m = 0$, and then θ_m approaches the vacuum value θ.

In the nonresonant channel, movement of the angle θ_m is simpler. Under the same variation of V_g as above it starts from $\theta_m = \theta_g$ and ends up with θ without crossing the points of zero mixing and resonance irrespective of the relative magnitudes of θ_g and θ.

Suppose that the initial potential (the potential at the production point) is much larger than the one at resonance, $\Phi_i \gg \Phi_R$, and the final potential is much smaller than the value at resonance, $\Phi_f \ll \Phi_R$. In this case the transition probability in the adiabatic approximation (21) becomes:

$$P_a = \frac{1}{2} \left(1 \pm \cos 2\theta_g \cos 2\theta\right) , \tag{22}$$

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where the plus sign is for the resonance channels ($\Delta m^2 \Delta f > 0$), and the minus sign is for the non-resonant channels ($\Delta m^2 \Delta f < 0$).

Let us mark one interesting feature related to zero mixing at $V_g = V_g^0$ (13). If the initial (final) potential is such that $V_g = V_g^0$ for $\theta_g < \theta$ ($\theta_g > \theta$), then the transition probability in the resonant channel reduces to $P = \cos^2 \theta$ ($P = \cos^2 \theta_g$) as in the case of flavor conversion in matter.

Let us discuss finally effects of possible complex phases in the Hamiltonian (11). After redefinition of the neutrino wave function only one phase survive which can be put in the gravitational term of the Hamiltonian (11) [12]:

$$
V_g \begin{pmatrix} -c_g & s_g e^{-i2\alpha} \\ s_g e^{i2\alpha} & c_g \end{pmatrix} . \tag{23}
$$

The angle α is the relative phase of the vacuum and gravitational contributions to the mixing (nondiagonal elements), and it is present when both of these contributions exist. Let us consider physical effects of the phase.

The Hamiltonian (11) can be rewritten as

$$
H(x) = \begin{pmatrix} -c\delta - V_g c_g & \rho_\alpha e^{-i2\psi} \\ \rho_\alpha e^{i2\psi} & c\delta + V_g c_g \end{pmatrix} , \tag{24}
$$

where

$$
\rho_\alpha = \left| \delta s + V_g s_g e^{-i2\alpha} \right| = V_g s_g \sqrt{\xi^2 + 2\xi \cos 2\alpha + 1} , \tag{25}
$$

with $\xi \equiv \delta s / V_g s_g$, and

$$
\tan 2\psi = \frac{V_g s_g \sin 2\alpha}{\delta s + V_g s_g \cos 2\alpha} . \tag{26}
$$

Additional redefinition of the fields

$$
\nu'_\mu = e^{i\psi} \nu_\mu , \quad \nu'_\tau = e^{-i\psi} \nu_\tau, \tag{27}
$$

leads to the evolution equation for (ν'_μ, ν'_τ) with the Hamiltonian

$$H(x) = \begin{pmatrix} -c\delta - V_g c_g - \dot{\psi} & \rho_\alpha \\ \rho_\alpha & c\delta + V_g c_g + \dot{\psi} \end{pmatrix} \; , \tag{28}$$

where $\dot{\psi} \equiv d\psi/dr$.

From this we get the following consequences.

1). There is no complex phases in the Hamiltonian (28). Thus, the presence of the phase α does not lead to the $CP-$ or $T-$ violating effects in the two generation case, as it was expected from the beginning.

2). The resonance condition is modified:

$$c\delta + V_g c_g + \dot{\psi} = 0 \; . \tag{29}$$

The shift of the resonance, $\dot{\psi}$, is absent in the case of constant gravitational potential for which $\dot{\psi} = 0$. If the adiabaticity condition is fulfilled in the absence of α then the shift of the resonance position is negligibly small. Indeed, for a maximal phase $\sin^2 2\alpha = 1$, we find from (26) that $\dot{\psi} < \frac{\dot{V_g}}{4V_g}$ and then after suitable modification including the effect of phase, the adiabaticity condition leads to $\dot{\psi} \ll V_g s_g$. The latter condition means that the shift of the resonance is much smaller than the width of the resonance.

3). Nonzero phase α modifies the dependence of the mixing parameter $\sin^2 2\theta_m$ on V_g. The mixing is proportional to the nondiagonal element of the Hamiltonian (28) ρ_α. For $\xi \gg 1$ or $\xi \ll 1$ it follows from (25) that $\rho_\alpha \approx \rho_0$, where ρ_0 corresponds to the case of zero α. That is, the effect of α is small when the gravity or vacuum effect dominate.

The strongest influence of α is for $\xi = -1$ which corresponds to the point of zero mixing: $V_g = V_0$. In fact, the phase remove zero mixing at V_0.

4). If $\xi \cos 2\theta > 0$, then $\rho_\alpha < \rho_0$; the phase leads to decrease of mixing and to narrower resonance peak. For $\xi \cos 2\theta < 0$ we get $\rho_\alpha > \rho_0$, i.e., the phase enhances mixing.

5). The presence of phase affects also the adiabaticity condition. However, it can be shown that if the adiabaticity is fulfilled at $\alpha = 0$, this influence is weak apart from some exceptional values of the phase.

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Thus we see that in many physically interesting situation the role of the complex phase is quite small even if $\alpha \sim O(1)$, and in what follows we will discuss the case $\alpha = 0$.

IV. FLAVOR TRANSITION OF THE AGN NEUTRINOS

Let us consider the flavor transitions of neutrinos from the AGN using the results of Sec. III. Following the scenarios described in [1], we assume that neutrinos are produced within the region located at the distance $R_e = (10 - 100)R_S$ from the center of AGN, where R_S is the Schwarzschild radius: $R_S \simeq 3 \times 10^{11}(M_{AGN}/10^8 M_\odot)$ m. For radii larger than the neutrino production point we may use the Keplerian approximation for the potential of the AGN. The total potential probed by neutrinos on the way to the Earth is

$$\Phi(r) \approx \Phi_{AGN}(r) + \Phi_{IG} = \Phi_{AGN}^0 \left(\frac{R_e}{r}\right) + \Phi_{IG} . \tag{30}$$

Here,

$$\Phi_{AGN}^0 \simeq -5 \times 10^{-3} \left(\frac{M_{AGN}}{10^8 M_\odot}\right) \tag{31}$$

is the AGN potential at the neutrino production point and, for simplicity, we take the potential in the intergalactic space to be constant: $\Phi_{IG} = 10^{-5}$. Therefore, at the neutrino production point the potential Φ_{AGN}^0 dominates over the supercluster and the galactic potentials. For AGN located at 100 Mpc from us it is about 3 and 7 orders of magnitude larger than the potentials of the great attractor and the Milky Way Galaxy, respectively. In what follows we neglect the potential of our Galaxy.

For fixed Δf the dependence of the transition probability on the neutrino energy (or $E^2/\Delta m^2$) is the following. For small energies the mass-induced vacuum oscillation effect dominates and

$$P \approx \frac{1}{2}\sin^2 2\theta \quad \text{for} \quad \frac{E^2}{\Delta m^2} < \frac{1}{2\Phi_{AGN}^0 \Delta f} . \tag{32}$$

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For larger energies the transition probability equals P_a in (22), if the adiabaticity condition is fulfilled:

$$P \approx P_a \quad \text{for} \quad \frac{E^2}{\Delta m^2} > \frac{1}{2\Phi^0_{AGN}\Delta f} \ . \tag{33}$$

Moreover, in the region where AGN potential dominates over intergalactic potential the resonance conversion takes place. This corresponds to

$$\frac{1}{2\Phi^0_{AGN}\Delta f} < \frac{E^2}{\Delta m^2} < \frac{1}{2\Phi_{IG}\Delta f} \ . \tag{34}$$

For these energies the transition probability is larger than $1/2$, and can be close to 1. The weaker the potential of the supercluster the wider the resonance-effective region of parameters. Moreover, if the $\Phi_{IG} < 10^{-7}$ the resonance conversion may take place in the gravitational field of our Galaxy too.

For higher energies the mass splitting can be neglected and the dominant effect is the one due to gravitationally induced oscillations:

$$P \approx \frac{1}{2}\sin^2 2\theta_g \ \ \text{for} \ \ \frac{E^2}{\Delta m^2} \gg \frac{1}{2\Phi_{IG}\Delta f} \ . \tag{35}$$

Once the adiabaticity condition is satisfied in the AGN gravitational field the transition probability (22) applies also to the nonresonant channels.

The resonance flavor conversion is effective thanks to the fact that the mass of the AGN is concentrated mainly in a small region of space ($\sim 10^{-5}$ kpc) of the central black hole. Since our Galaxy is more massive than the typical AGN one might expect that neutrinos converted at the AGN could be reconverted by our Galaxy's gravitational field. It does not occur for $\Phi_{IG} \sim 10^{-5}$: the gravitational potential of the Galaxy is at most $\Phi_G \sim 10^{-7}$ because its mass is distributed over the ~ 10 kpc region.

Let us find the sensitivity to Δf assuming that future underwater/ice experiments will be sensitive to the flavor transition with probability as small as in (1) [2]. We can distinguish three ranges of Δf corresponding to three energy regions defined in (32), (34) and (35).

Region I:

$$\Delta f < \Delta f_{AGN} \ , \tag{36}$$

where

$$\Delta f_{AGN} = 10^{-30} \times \left(\frac{\Delta m^2}{10^{-2}\text{eV}^2}\right)\left(\frac{\bar{E}}{1\text{PeV}}\right)^{-2}\left(\frac{R_e}{100R_S}\right)\left(\frac{M_{AGN}}{10^8 M_\odot}\right)^{-1} . \tag{37}$$

Here $\bar{E}$ is the average energy of the detected neutrinos. The gravity does not play any role and the effect of mass-induced vacuum oscillation dominates.

Region II:

$$\Delta f_{AGN} < \Delta f < \Delta f_{IG} \ , \tag{38}$$

where

$$\Delta f_{IG} = 5 \cdot 10^{-28} \times \left(\frac{\Delta m^2}{10^{-2}eV^2}\right)\left(\frac{\bar{E}}{1\text{PeV}}\right)^{-2}\left(\frac{10^{-5}}{\Phi_{IG}}\right) . \tag{39}$$

In this region neutrinos undergo resonance conversion driven by the potential of the AGN, and the transition probability can be close to 1, if both θ and θ_g are small.

Suppose that the factor $(\tan 2\theta - \tan 2\theta_g)^2 \cos 2\theta_g$ in the adiabaticity condition (20) is of order unity. Then the adiabaticity condition is satisfied for $\Delta f \gg 2 \times 10^{-33}$. Comparing this inequality with (39) we find that for $\Delta m^2 \gtrsim 10^{-4}$ eV^2 the adiabaticity holds in whole region where the resonance condition is met. If 10^{-7} eV$^2 \lesssim \Delta m^2 \lesssim 10^{-4}$ eV^2 the adiabaticity region partially overlaps with the resonance region. If $\Delta m^2 \lesssim 10^{-7}$ eV^2 there is no overlapping between the two regions and the resonance conversion is not efficient.

Region III:

$$\Delta f \gg \Delta f_{IG} . \tag{40}$$

Here gravitational effect dominates and neutrinos oscillate due to the VEP. The probability converges to $P = \frac{1}{2}\sin^2 2\theta_g$.

V. OBSERVATIONAL SIGNATURE OF GRAVITY EFFECTS

The underwater/ice detectors will be able to measure the ratio of number of events induced by ν_τ and ν_μ, ν_τ/ν_μ, as well as the ratio of ν_e and ν_μ events, ν_e/ν_μ.

If neutrinos are massless or degenerate, the excess of the τ neutrino signal in underwater installations will testify for the neutrino flavor transition induced by the VEP. As we discussed in sect. 2 the appropriate range of parameters is $\Delta f > 10^{-41}$ and $\sin^2 2\theta_g \gtrsim 0.01$.

How can one prove this?

Evidently, the anisotropy of ν_τ/ν_μ and ν_e/ν_μ ratios correlated to the position of the supercluster and/or the center of our Galaxy could be a signature of the gravitational origin of the excess. Let us compare the gravitational effect in the direction towards and away from the supercluster. Consider the AGN's at the distance R from the center of supercluster in the cone directed from the Earth toward and outward the supercluster. Neutrinos from such AGN will acquire, on the way to the Earth, the gravitational phases

$$\phi_g^{toward} = \Delta\phi + \phi \ , \quad \phi_g^{away} = \phi \ , \tag{41}$$

where

$$\Delta\phi = \Delta f E M_{sc} G \left[2 \ln \frac{d}{r_0} + 1 \right] \ , \quad \phi = \Delta f E M_{sc} G \left[\ln \frac{R}{d} \right] \ . \tag{42}$$

Here d is the distance from the Earth to the supercluster and r_0 is the size of the supercluster. The difference of phases is described by $\Delta\phi$. The effect is determined by integration over R. (If we neglect the expansion of the Universe the flux of neutrinos from different R will be the same in the case of uniform distribution of the AGN). The anisotropy will be observable provided that the following conditions are met:

1). $\Delta\phi \sim 1$, i.e. the phase should be large enough.

2). $\phi(R = ct_u) < \Delta\phi$, where t_u is the age of the Universe. This condition ensures the absence of the spatial averaging.

3). Events within sufficiently small energy interval $\Delta E < E$ are selected to avoid the averaging over the energy.

For $d = 50$ Mpc, $r_0 = 1$ Mpc we get $\Delta\phi/\phi \simeq 2$. It means that for the supercluster the conditions 1). and 2). can be fulfilled for certain values of E and Δf. However, observation of the asymmetry will require high statistics, since one should select the events within small solid angle, $\sim (r_0/d)^2$, and an energy-cut is required.

The conditions 1). and 2). are not satisfied for our Galaxy as the source of the asymmetry.

If an excess of ν_τ events will not be found, then this will allow one to exclude the region of parameters, $\sin^2 2\theta_g > 10^{-2}$ and $\Delta f > \Delta f_{AGN}$, where in the expression (37) for Δf_{AGN} one should take as Δm^2 the upper experimental bound on neutrino mass difference.

As we have discussed in Sec. III most probably neutrinos are massive and mixed, and moreover, there is a good chance that forthcoming experiments will measure Δm^2 and $\sin^2 2\theta$. In the case of massive neutrinos a signature of the gravity effect is quite different from that of the massless case. One should look for the deviation of the observed ratio ν_τ/ν_μ's (as well as ν_e/ν_μ) from that stipulated by vacuum oscillations. The gravity induced mixing can both suppress and enhance ν_τ-signal.

If Δf is in Region II (38) the gravitational MSW effect occurs in the resonance channel. The resonance conversion of ν_μ to ν_τ gives rise to a large transition probability $P > 1/2$. Therefore, the observation of the ratio ν_τ/ν_μ's larger than unity would provide an evidence for the gravitational MSW effect.

In the nonresonant channel the gravity effect is described by the transition probability (22) with minus sign. For $\theta_g > \theta$ ($\theta_g < \theta$), the gravity effect enhances (suppresses) the transition. The modification is maximal if the vacuum angle θ is small and the gravity angle is large, $\theta_g \sim \pi/4$.

If Δf falls into Region III (40) the signal would mimic the one due to vacuum flavor oscillation. But, since we assume that the masses and vacuum mixing angles will be determined by future experiments, the difference between θ_g and θ should show up in the measured ratio of ν_τ to ν_μ.

Let us discuss experimental signatures and estimate the sensitivity regions for the VEP parameters for two probable scenarios of neutrino masses and mixing. They are suggested by some experimental results at hand and will be checked by forthcoming experiments.

1. Suppose the heaviest neutrino (which practically coincides with ν_τ) has the mass in the

cosmologically interesting region: $m_3 = (3 - 7)$ eV, so that $\Delta m^2 = (10 - 50)$ eV^2 in the case of mass hierarchy. For these values of mass the present experimental bound on mixing angle is already $\sin^2 2\theta < 5 \cdot 10^{-3}$, i.e. below the sensitivity limit (1). We will assume also that mixing with electron neutrinos is negligibly small.

In this case the effect of mass-induced vacuum oscillation can be neglected. The result for the ratio $\nu_\tau/\nu_\mu > 0.01$ in the underwater/ice detectors would then be an indicator of the VEP mechanism of flavor conversion. Observation of large value of the ratio: $\nu_\tau/\nu_\mu \gtrsim 1$, would provide a clear signature for the gravitational MSW effect. From the resonance condition we find, assuming the typical values for the parameters in (37) and (39), the required region for Δf:

$$\Delta f = (10^{-27} - 5 \times 10^{-25}) \left(\frac{\Delta m^2}{10 \text{ eV}^2} \right) . \tag{43}$$

Then, for fixed Δf, the adiabaticity condition gives the bound on mixing angles

$$(\tan 2\theta - \tan 2\theta_g)^2 \cos 2\theta_g \gg 5 \times 10^{-9} \left(\frac{\Delta f}{5 \times 10^{-25}} \right)^{-1} \left(\frac{\Delta m^2}{10 \text{ eV}^2} \right)^{-1} . \tag{44}$$

If lepton mixing is similar to the quark mixing, then one expects $\sin^2 2\theta > 10^{-3}$ for $\nu_\tau - \nu_\mu$. In this case the adiabaticity condition is satisfied by vacuum mixing alone and the gravitational angle can be arbitrarily small.

Non-observation of such a signal will allow one to exclude the whole region of parameters in (43) and θ_g being not too close to θ.

Observation of a moderately-large ratio, $0.01 < \nu_\tau/\nu_\mu < 1$ would be the signal for one of the following three possibilities; the nonadiabatic gravitational MSW mechanism (in resonant channel), the nonresonant adiabatic conversion with transition probability $P_a \approx \sin^2 \theta_g$ (in nonresonant channel), or oscillations in the intergalactic gravitational field with $\sin^2 2\theta_g > 0.01$. In the first case, the parameter Δf should take the value around (43) and $\theta_g \approx \theta$ to violate the adiabaticity which is well satisfied by vacuum mixing alone. In the other two cases, the region $\sin^2 2\theta_g \gtrsim 0.01$ will be probed.

2. Suppose that the heaviest neutrino has a mass $m_3 \sim 0.1$ eV and $\nu_\mu - \nu_\tau$ mixing is large: $\sin^2 2\theta \sim 0.5 - 1$, so that $\nu_\mu - \nu_\tau$ oscillations solve the atmospheric neutrino problem. As the result of the vacuum oscillations one predicts the ratio $\nu_\tau / \nu_\mu \leq 1$ for neutrinos from AGN. The observation of larger ratio, $\nu_\tau / \nu_\mu \gtrsim 1$, will be a signature for the gravitational MSW effect. In this case the VEP parameter should be in the interval

$$\Delta f = (0.3 - 1) \times 10^{-30} - 5 \times 10^{-28} , \tag{45}$$

assuming again the typical values of parameters in the parentheses in (37) and (39). The adiabaticity condition is satisfied by large vacuum mixing angle, and the angle θ_g can be arbitrarily small.

In the nonresonant channel the gravitational effect manifests in a different manner. If Δf is in Region II (38), i.e., $\Delta f > 10^{-30}$, and the gravitational mixing angle θ_g is smaller than the vacuum angle θ, the gravitational effect suppresses neutrino transition. Namely, the ν_τ signal will be smaller than the one expected for mass-induced vacuum oscillations. Using (22) with $\theta_g \ll 1$ we find transition probability $P = \sin^2 \theta$ instead of $P = \frac{1}{2} \sin^2 2\theta$ in the absence of the VEP. In such a way the transition probability is reduced by 40% and 30% for $\sin^2 2\theta = 0.6$ and 0.8, respectively.

In Region III (40) the averaged transition probability equals $P = \frac{1}{2} \sin^2 2\theta_g$ and for small θ_g suppression can be much stronger, so that ν_τ signal will not be observable at all.

VI. CONCLUSION

1. Cosmological distances (~ 100 Mpc) and ultrahigh energies (~ 1 PeV) of neutrinos from the AGN open unique possibility to improve an accuracy of testing the equivalence principle by 12 - 21 orders of magnitude.

2. For massless neutrinos VEP can induce the oscillations $\nu_\mu - \nu_\tau$ of cosmic neutrinos which may lead to observable ν_τ signals in the large underwater/ice installations. The sensitivity

to the parameters of the VEP can be estimated as $\Delta f \gtrsim 10^{-41}$ and $\sin^2 2\theta_g > 2 \cdot 10^{-2}$. In the case of the nonaveraged oscillations (Δf at the lower bound) one can expect an anisotropy of the ν_τ signal correlated to the position of the Great Attractor.

3. In the case of massive neutrinos the gravitational effects due to VEP can modify the result of the vacuum oscillations. For certain values of the parameters neutrinos may undergo the resonance flavor conversion driven by the gravitational potential of AGN (or of our Galaxy, if the intergalactic potential is sufficiently weak). The gravitational effects become important if $\Delta f \gtrsim \Delta f_{AGN} \sim 10^{-28}(\Delta m^2/1 \text{ eV}^2)$. For $\Delta f \sim (1 - 10^3)\Delta f_{AGN}$ one may expect almost complete transition of ν_μ to ν_τ due to the resonance conversion. A strong observable effect may exist for arbitrarily small $\sin^2 2\theta_g$. For $\Delta f \gg (1 - 10^3)\Delta f_{AGN}$ neutrinos undergo the gravity induced oscillations and vacuum mixing effect can be neglected.

4. The VEP effects can be identified if the gravitational mixing differs appreciably from vacuum mixing. In general, the VEP effect will manifest itself as deviation of the observed ratios ν_τ/ν_μ and ν_μ/ν_e from those stipulated by vacuum oscillations. Of course, the latter can be predictable only if neutrino masses and mixing are determined by forthcoming neutrino experiments. For small vacuum mixing the VEP can enhance the $\nu_\mu - \nu_\tau$ transition and the ν_τ-signal. On the contrary, for large vacuum mixing the VEP can lead to suppression of the ν_τ-signal. The ratio $\nu_\tau/\nu_\mu > 1$ is the clear signature of the resonance flavor conversion.

5. If deviation from vacuum oscillation effects will not be found, then one will be able to exclude very large new region of the VEP - parameters.

VII. ACKNOWLEDGEMENTS

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